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Cosmological Redshift Distortion of Correlation Functions as a Probe of the Density Parameter and the Cosmological Constant

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Abstract

We propose cosmological redshift-space distortion of correlation functions of galaxies and quasars as a probe of both the density parameter Ω_0 and the cosmological constant λ_0. In particular, we show that redshift-space distortion of quasar correlation functions at $z \sim 2$ can in principle set a constraint on the value of λ_0. This is in contrast This is in contrast to the popular analysis of galaxy correlation functions in redshift space which basically determines $\Omega_0^{0.6}/b$, where b is the bias parameter, but is insensitive to λ_0. For specific applications, we present redshift-space distortion of correlation functions both in cold dark matter models and in power-law correlation function models, and discuss the extent to which one can discriminate between the different λ_0 models.

Subject headings: cosmology: theory — large-scale structure of the universe — methods: statistical

1 Introduction

Redshift-space distortion of galaxy two-point correlation functions is known as a powerful tool in estimating the cosmological density parameter Ω_0; on nonlinear scales Davis & Peebles (1983) computed the relative peculiar velocity dispersions of pair of galaxies around $1h^{-1}$Mpc from the anisotropies in the correlation functions of the CfA1 galaxy redshift survey, and then concluded that $\Omega_0 = 0.20e^{\pm 0.4}$ (also see Mo, Jing & Börner 1993; Suto 1993; Ratcliffe et al. 1996). In linear theory Kaiser (1987) showed that the peculiar velocity field systematically distorts the correlation function observed in redshift space; the averaged redshift-space correlation function $\xi^{(s)}(x)$ of galaxies is related to its real-space counter part $\xi^{(r)}(x)$ as

$$\xi^{(s)}(x) = \left(1+\frac{2}{3}\beta+\frac{1}{5}\beta^2\right)\xi^{(r)}(x), \tag{1}$$

$$\beta = \frac{f(z=0)}{b} \sim \frac{1}{b}\left[\Omega_0^{0.6}+\frac{\lambda_0}{70}\left(1+\frac{\Omega_0}{2}\right)\right], \tag{2}$$

where b is the bias parameter, $f(z=0)$ is the logarithmic derivative of the linear growth rate with respect to the scale factor a (eq.[15] below) at $z=0$. As is clear from the empirical fitting formula in the second line by Lahav et al. (1991), however, this formula depends on Ω_0 but is practically insensitive to the cosmological constant λ_0.

At higher redshift $z \gtrsim 1$, another anisotropy due to the geometrical effect of the spatial curvature becomes important. We derive a formula for *the cosmological redshift-space distortion* in linear theory. This formula turns out to be a straightforward generalization of that derived by Hamilton (1992) for $z=0$. It provides a promising method to estimate both Ω_0 *and* λ_0 from future redshift surveys of galaxies and quasars including the Sloan Digital Sky Survey (SDSS) and the 2dF. A very similar idea was put forward independently by Ballinger, Peacock and Heavens (1996) in which they considered the anisotropy of the power spectrum while we developed a formulation in terms of two-point correlation functions. We outline the derivation in §2, and present some examples of results in cold dark matter (CDM) models and in power-law correlation function models.

2 Cosmological redshift distortion

Throughout the present analysis, we assume a standard Robertson – Walker metric of the form:

$$ds^2 = -dt^2 + a(t)^2\{d\chi^2 + S(\chi)^2[d\theta^2+\sin^2\theta d\phi^2]\}, \tag{3}$$

where $S(\chi)$ is determined by the sign of the spatial curvature K as

$$S(\chi) = \begin{cases} \sin(\sqrt{K}\chi)/\sqrt{K} & (K>0) \\ \chi & (K=0) \\ \sinh(\sqrt{-K}\chi)/\sqrt{-K} & (K<0) \end{cases} \tag{4}$$

In our notation, K is written in terms of the scale factor at present a_0 and the Hubble constant H_0 as

$$K = a_0^2 H_0^2 (\Omega_0 + \lambda_0 - 1). \tag{5}$$

The radial distance $\chi(z)$ is computed by

$$\chi(z) = \int_t^{t_0} \frac{dt}{a(t)} = \frac{1}{a_0} \int_0^z \frac{dz}{H(z)}, \tag{6}$$

where we introduce the Hubble parameter at redshift z:

$$H(z) = H_0 \sqrt{\Omega_0 (1+z)^3 + (1 - \Omega_0 - \lambda_0)(1+z)^2 + \lambda_0}. \tag{7}$$

Consider a pair of galaxies or quasars located at redshifts z_1 and z_2. If both the redshift difference $\delta z \equiv z_1 - z_2$ and the angular separation of the pair $\delta\theta$ are much less than unity, the comoving separations of the pair parallel and perpendicular to the line-of-sight direction, $x_\parallel$ and $x_\perp$, are given by

$$x_\parallel(z) = \frac{d\chi(z)}{dz}\delta z = \frac{c_\parallel \delta z}{a_0 H_0}, \quad x_\perp(z) = S(\chi(z))\delta\theta = \frac{c_\perp z \delta\theta}{a_0 H_0}, \tag{8}$$

where $c_\parallel(z) = H_0/H(z)$, $c_\perp(z) = a_0 H_0 S(\chi(z))/z$ and $z \equiv (z_1 + z_2)/2$. Thus their ratio becomes

$$\frac{x_\parallel(z)}{x_\perp(z)} = \frac{c_\parallel(z)}{c_\perp(z)} \frac{\delta z}{z\delta\theta} \equiv \eta(z) \frac{\delta z}{z\delta\theta}. \tag{9}$$

Since $x_\parallel/x_\perp$ should approach $\delta z/(z\delta\theta)$ for $z \ll 1$, $\eta(z)$ can be regarded to represent the correction factor for the *cosmological redshift distortion* (upper panel of Fig.1). This was first pointed out in an explicit form by Alcock & Paczyński (1979).

Although $\eta(z)$ is a potentially sensitive probe of Ω_0 and especially λ_0, this is not directly observable unless one has an independent estimate of the ratio $x_\parallel/x_\perp$. Therefore Alcock & Paczyński (1979) proposed to apply the result to intrinsically spherical structure resulting from gravitational clustering. More recently Ryden (1995) proposed to apply the test to statistically spherical voids around $z \lesssim 0.5$. It is not likely, however, that the intrinsic shape of the objects formed via gravitational clustering is spherical even in a statistical sense. Thus their proposal would be seriously contaminated by the aspherical shape distribution of the objects considered. In what follows, we propose to use the clustering of quasars and galaxies in z, which should be safely assumed to be statistically isotropic, in estimating $\eta(z)$. Also we take account of the distortion due to the peculiar velocity field using linear theory.

If an object located at redshift z_1 has a peculiar recession velocity $v_{\parallel,1}$ with respect to us, we actually *observe* its redshift as

$$1 + z_{\mathrm{obs},1} = (1 + z_1)(1 + v_{\parallel,1} - v_{\parallel,0}) \tag{10}$$

where $v_{\parallel,0}$ is our (observer's) peculiar velocity, and we assume that all peculiar velocities are non-relativistic. Let us consider a sample of galaxies or quasars located in the range $z - dz \sim z + dz$ with $dz \ll z$ distribute over an angular radius comparable to the proper distance corresponding to dz. Then we set the locally Euclidean coordinates in real space (comoving), $\boldsymbol{x}$, and in redshift space, $\boldsymbol{s}$, which share the same origin at redshift z. Since we assume that these coordinates are far from us, we can use the distant-observer approximation. If we take the third axis along the line of sight, the above two coordinates are related to each other as

$$s_1 = \frac{x_1}{c_\perp(z)}, \quad s_2 = \frac{x_2}{c_\perp(z)}, \tag{11}$$

$$s_3 = \frac{z_{\mathrm{obs},1} - z}{H_0} \simeq \frac{1}{c_\parallel(z)} \left[x_3 + \frac{1+z}{H(z)} (v_\parallel - v_{0\parallel}) \right]. \tag{12}$$

Note that in the last expression, we assume that x_3 is sufficiently small and replace $1 + z + Hx_3$ by $1 + z$. The number densities of objects in these coordinates are related by the Jacobian of the above transformation. In linear order of density contrast and peculiar velocity, we obtain

$$\delta^{(s)}(\boldsymbol{s}(\boldsymbol{x})) = \delta^{(r)}(\boldsymbol{x}) - \frac{1+z}{H(z)} \partial_3 v_\parallel. \tag{13}$$

According to linear theory, the peculiar velocity is related to the *mass* density fluctuation δ_{m} (e.g., Peebles 1993) as

$$v_\parallel(\boldsymbol{x}) = -\frac{H(z)}{1+z} f(z) \partial_3 \triangle^{-1} \delta_{\mathrm{m}}(\boldsymbol{x}), \tag{14}$$

where $\triangle^{-1}$ is the inverse Laplacian,

$$f(z) \equiv \frac{d \ln D(z)}{d \ln a} \simeq \Omega(z)^{0.6} + \frac{\lambda(z)}{70} \left(1 + \frac{\Omega(z)}{2} \right), \tag{15}$$

and the linear growth rate $D(z)$ (normalized as $D(z) = 1/(1+z)$ for $z \to \infty$) is

$$D(z) = \frac{5 \Omega_0 H_0^2}{2} H(z) \int_z^\infty \frac{1+z'}{H(z')^3} dz'. \tag{16}$$

In the above, $\Omega(z)$ and $\lambda(z)$ are the density parameter and the dimensionless cosmological constant at z:

$$\Omega(z) = \left[\frac{H_0}{H(z)} \right]^2 (1+z)^3 \Omega_0, \quad \lambda(z) = \left[\frac{H_0}{H(z)} \right]^2 \lambda_0. \tag{17}$$

Allowing for the possibility that the density contrast of *mass*, δ_{m}, differs from that of *objects* $\delta^{(r)}$, we assume linear biasing $\delta^{(r)} = b\, \delta_{\mathrm{m}}$, where $b(z)$ is a bias factor which now depends on z. Then equation (13) simply generalizes Kaiser (1987)'s result to $z \neq 0$:

$$\delta^{(s)}(\boldsymbol{s}(\boldsymbol{x})) = \int \frac{d^3k}{(2\pi)^3} \left[1 + \beta(z) \frac{k_3{}^2}{k^2} \right] e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \tilde{\delta}^{(r)}(\boldsymbol{k}), \tag{18}$$

where $\beta(z) = f(z)/b(z)$, and $\widetilde{\delta}^{(r)}$ is the Fourier transform of the density contrast. From equation (18), we derive the following relation for the two-point correlation function at $z \neq 0$ which also generalizes the formula of Hamilton (1992) at $z = 0$:

$$\xi^{(s)}(s_\perp, s_\parallel) = \left(1 + \frac{2}{3}\beta(z) + \frac{1}{5}[\beta(z)]^2\right)\xi_0(x)P_0(\mu) - \left(\frac{4}{3}\beta(z) + \frac{4}{7}[\beta(z)]^2\right)\xi_2(x)P_2(\mu)$$
$$+\frac{8}{35}[\beta(z)]^2\xi_4(x)P_4(\mu), \tag{19}$$

where $x \equiv \sqrt{{c_\parallel}^2{s_\parallel}^2 + {c_\perp}^2{s_\perp}^2}$, $\mu \equiv c_\parallel s_\parallel / x$ ($s_\parallel = s_3$ and $s_\perp^2 = s_1^2 + s_2^2$), P_n's are the Legendre polynomials, and

$$\begin{aligned} \xi_{2l}(x) &\equiv \frac{1}{2\pi^2}\int_0^\infty dk k^2 j_{2l}(kx) P(k;z) \\ &= \frac{(-1)^l}{x^{2l+1}}\left(\int_0^x x dx\right)^l x^{2l}\left(\frac{d}{dx}\frac{1}{x}\right)^l x\xi^{(r)}(x;z). \end{aligned} \tag{20}$$

Again in linear theory and for linear biasing, the power spectrum $P(k;z)$ of objects at z is related to that of *mass* $P^{(m)}(k)$ of matter at $z = 0$ as

$$P(k;z) = [b(z)]^2\left(\frac{D(z)}{D(0)}\right)^2 P^{(m)}(k), \tag{21}$$

3 Simple model predictions

The lower panel in Figure 1 plots $f(z)$. Here and in what follows, we use the fitting formula (15) for $f(z)$ whose mean error is 2 percent. As expected from equation (2), $f(z = 0)$ is insensitive to the value of λ_0 and basically determined by Ω_0 only. At higher redshifts, however, $f(z)$ becomes sensitive to λ_0 as pointed out earlier by Lahav et al (1991), especially if $\Omega_0 \ll 1$. Therefore the behavior of $f(z)$ at low and high z is a potentially good discriminator of Ω_0 and λ_0, respectively. What we propose here is that a careful analysis of the redshift distortion in correlation functions of galaxies at low redshifts and quasars at high redshifts can probe Ω_0 and λ_0 through $f(z)$ as well as $\eta(z)$.

For specific examples, we compute $\xi^{(s)}(s_\perp, s_\parallel)$ in linear theory applying equations (19) and (20) in CDM models with $H_0 = 70$ km$\cdot$s$^{-1}\cdot$Mpc^{-1} (Tanvir et al. 1995). The resulting contours are plotted in Figure 2. The four sets of values of Ω_0 and λ_0 are the same as adopted in Figure 1, and we adopt the COBE normalization (Sugiyama 1995; White and Scott 1995).

In order to quantify the cosmological redshift distortion in Figure 2, let us introduce the anisotropy parameter $\xi_\parallel^{(s)}(s)/\xi_\perp^{(s)}(s)$, where $\xi_\perp^{(s)}(s) \equiv \xi^{(s)}(s, 0)$ and $\xi_\parallel^{(s)}(s) \equiv \xi^{(s)}(0, s)$. The left four panels in Figure 3 show the anisotropy parameter against z in CDM models. Since the evolution of bias is largely unknown, we assume $b = 1$ and 2 just for definiteness. The effect of the evolution of bias (e.g., Fry 1996) will be considered elsewhere (Suto &

Matsubara 1996). This clearly exhibits the extent to which one can discriminate the different λ_0 models on the basis of the anisotropies in $\xi^{(s)}$ at high redshifts.

For comparison, let us consider a simple power-law model $\xi^{(r)}(x;z) = A(z)x^{-\gamma}$ ($\gamma > 0$). Then equation (19) reduces to

$$\frac{\xi^{(s)}(s_\perp, s_\parallel)}{\xi^{(r)}(\sqrt{c_\parallel^2 s_\parallel^2 + c_\perp^2 s_\perp^2}; z)} = 1 + \frac{2(1-\gamma\mu^2)}{3-\gamma}\beta(z) + \frac{\gamma(\gamma+2)\mu^4 - 6\gamma\mu^2 + 3}{(3-\gamma)(5-\gamma)}\beta(z)^2. \qquad (22)$$

In this case the anisotropy parameter is given by

$$\frac{\xi_\parallel^{(s)}(s)}{\xi_\perp^{(s)}(s)} = \eta(z)^{-\gamma}\frac{(3-\gamma)(5-\gamma) + 2(1-\gamma)(5-\gamma)\beta(z) + (3-\gamma)(1-\gamma)\beta(z)^2}{(3-\gamma)(5-\gamma) + 2(5-\gamma)\beta(z) + 3\beta(z)^2}, \qquad (23)$$

and is independent of the scale s.

The right four panels in Figure 3 show that the behavior of the anisotropy parameter is sensitive to the power-law index γ. This is partly because $\Omega_0 = 1$ CDM models which we consider are already nonlinear on $(10 \sim 20)h^{-1}$Mpc (Fig.2), and then the linear theory prediction is not reliable enough. On the other hand, $\Omega = 0.1$ CDM models are well described in linear theory on the scales of our interest $\gtrsim 1h^{-1}$Mpc. Since most observations do suggest that Ω_0 in our universe is around $(0.1 \sim 0.3)$ (e.g., Peebles 1993; Suto 1993; Ratcliffe et al. 1996), this is encouraging for our present result on the basis of linear theory. Incidentally, Figure 3 also implies that $\Omega = 0.1$ CDM models in the linear regime are well approximated by the power-law model with $\gamma \sim 1$. This is reasonable since the CDM spectral index of $P(k)$ is around -2 which corresponds to $\gamma \sim 1$.

4 Conclusions

Redshift-space distortion of galaxy correlation functions has attracted much attention as a tool to determine $\Omega_0^{0.6}/b$ (Kaiser 1987; Hamilton 1992). We obtained an expression to describe the cosmological redshift-space distortion at high z in linear theory taking proper account of the spatial curvature K. Then we showed that in principle this can discriminate the value of λ_0 via the z-dependence of the $\beta(z)$ and $\eta(z)$ parameters.

Recent analysis, for instance of the Durham/UKST galaxy redshift survey on the basis of eq.(1), yields $\Omega_0^{0.6}/b = 0.55 \pm 0.12$ (Ratcliffe et al. 1996) with ~ 2500 galaxies. Their Figure 4(a) clearly exhibits that the observational data at $z = 0$ are already statistically reliable for the direct quantitative comparison with the upper panels in our Figure 2, although we do not attempt it at this point. SDSS, for instance, is expected to have a million of galaxy redshifts up to $z \lesssim 0.2$ which would be able to determine $\Omega_0^{0.6}/b$ to better than 10 percent accuracy. Again upon completion of SDSS, $\sim 10^5$ quasar samples become available and the anisotropy in quasar correlation functions at $z \sim 2$ will put a constraint on λ_0, given Ω_0 determined from the galaxy correlation functions. The quasar luminosity function of Boyle, Shanks & Peterson (1988) predicts that the number of quasars per

π steradian brighter than 19 B magnitude is about 4500 either in $z = 0.9 \sim 1.1$ or in $z = 1.9 \sim 2.1$ (for the $\Omega_0 = 1$ and $\lambda_0 = 0$ model). Therefore we expect that the resulting statistics is even better than that obtained by Ratcliffe et al. (1996) if unknown evolution effects and other potential systematics interfere.

Although the linear theory which we used throughout the paper becomes less restrictive at higher redshifts, it is observationally easier to detect clustering features in the nonlinear regime. Thus the analysis of nonlinear redshift-space distortion is another important area for research (Suto & Suginohara 1991; Matsubara & Suto 1994; Suto & Matsubara 1994; Matsubara 1994). In addition, it is important to examine the possible systematic errors due to the finite volume size and the shape of the survey region. This can be best investigated by the direct analysis of the numerical simulations. This is partly considered by Ballinger, Peacock, & Heavens (1996) although in k-space. We plan to return to this in the later paper (Magira, Matsubara, & Suto 1996).

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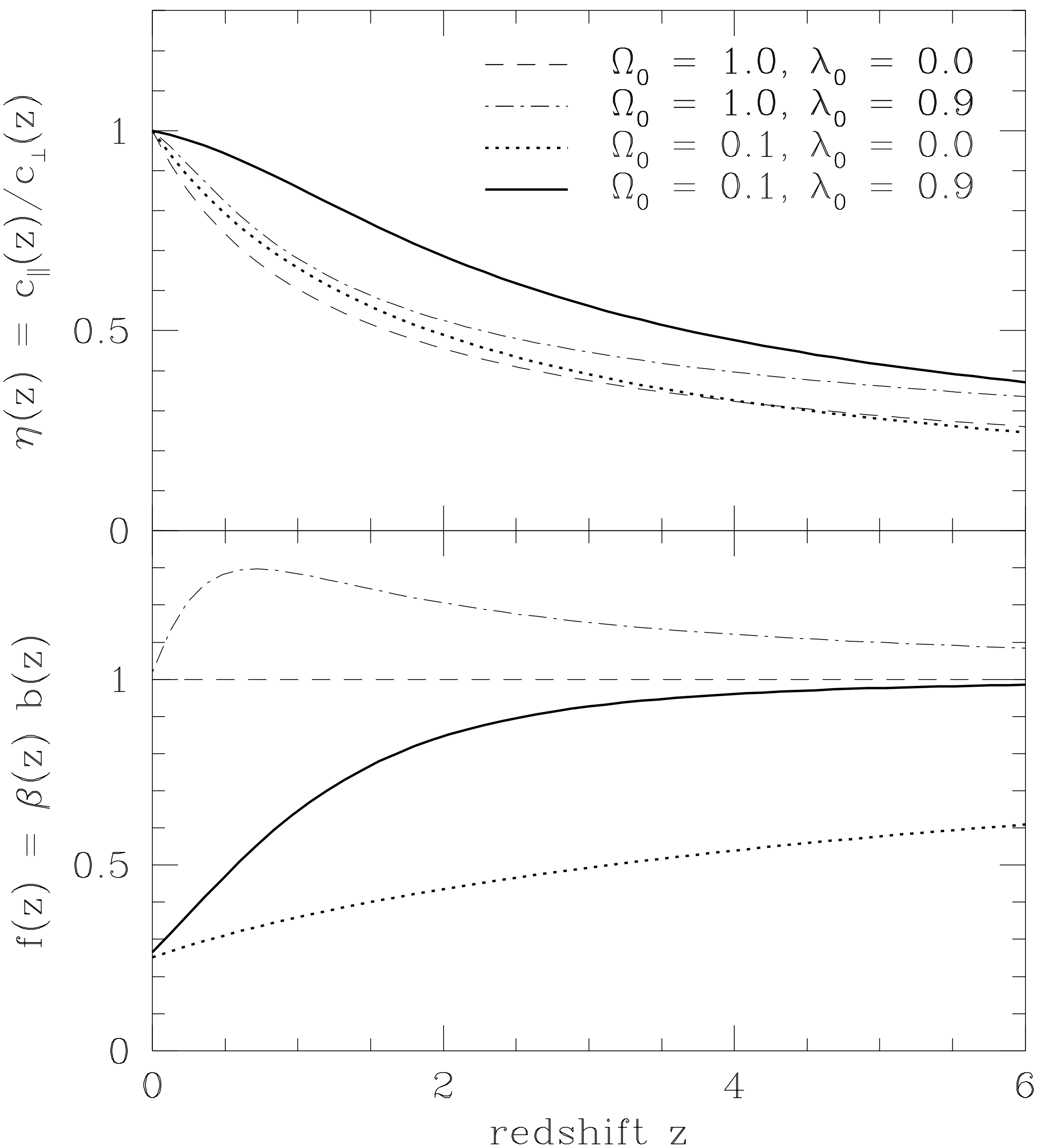
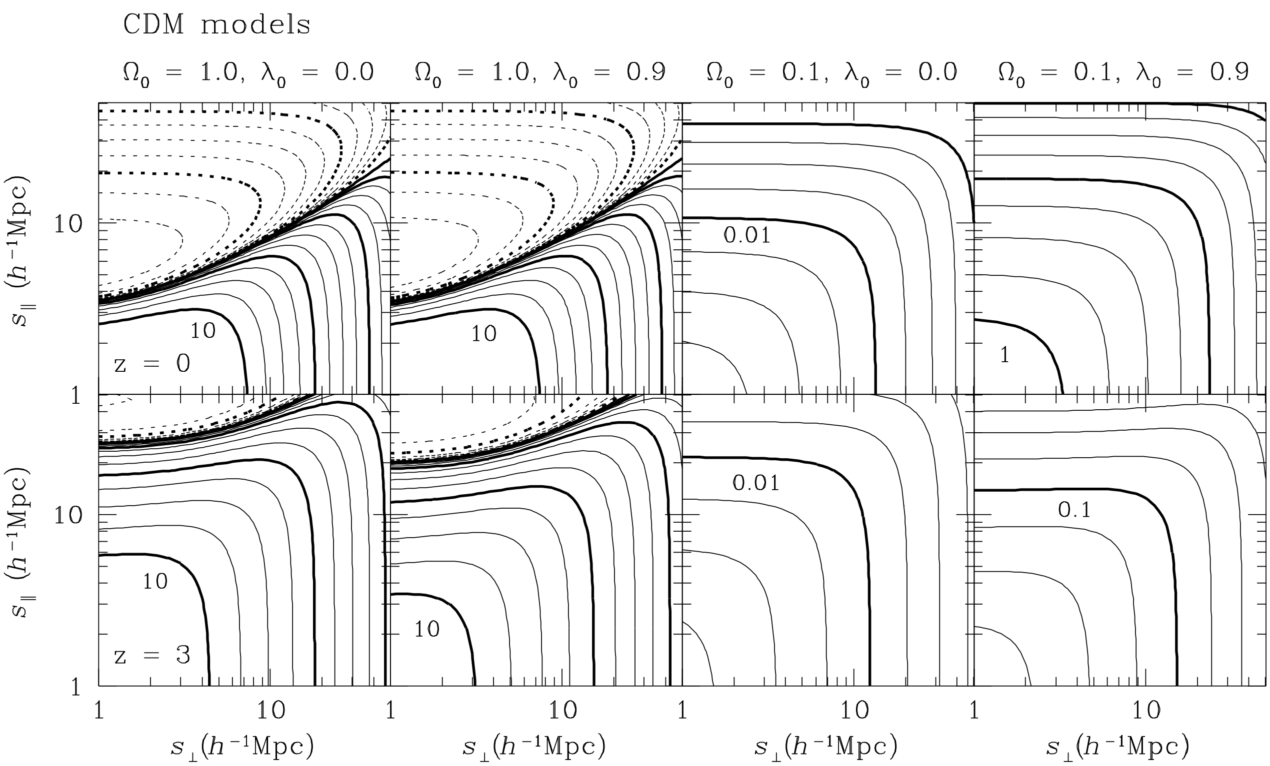
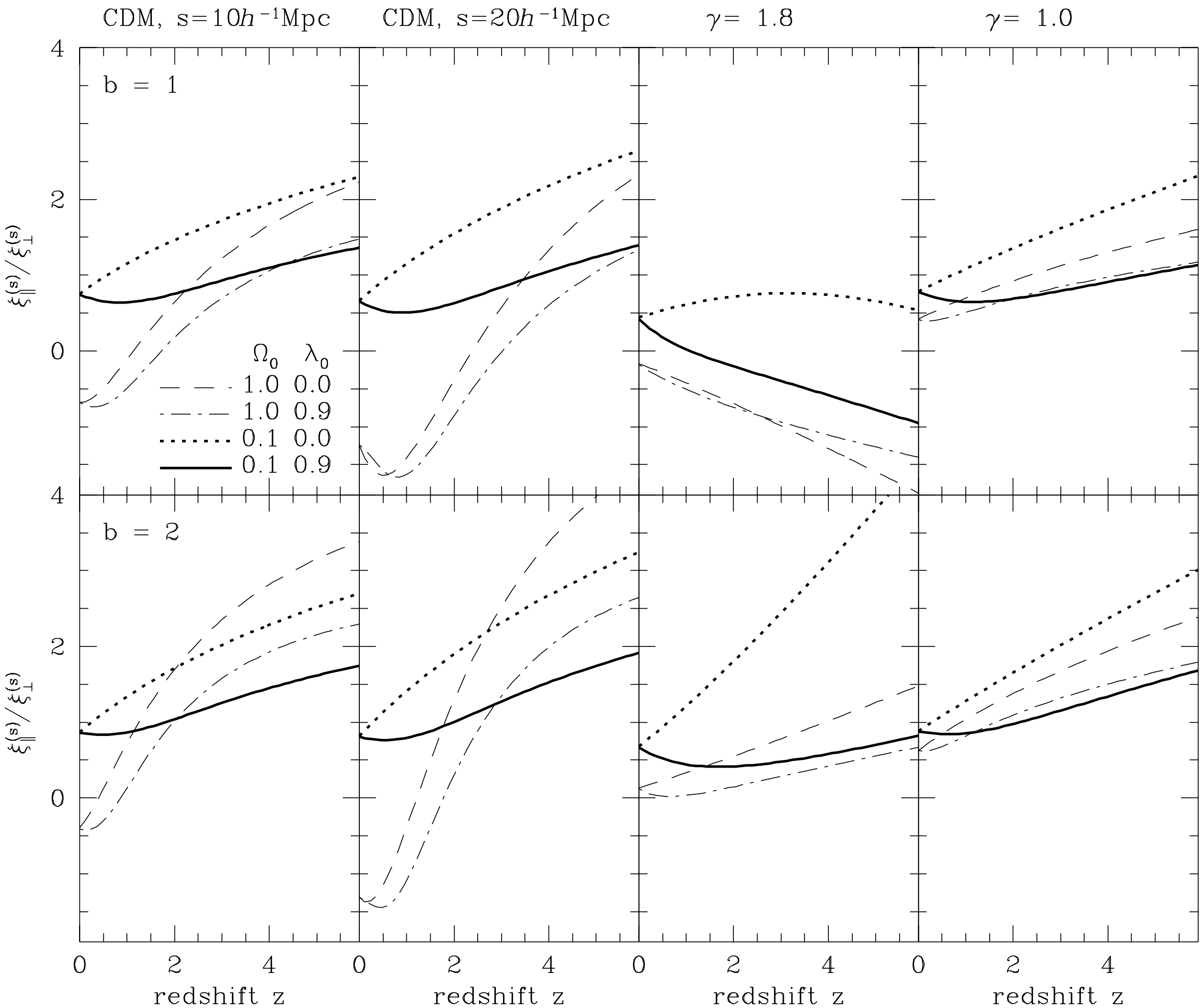
Figure 1 : $\eta(z) = c_{\parallel}(z)/c_{\perp}(z)$ (*upper panel*), and $f(z) = \beta(z)b(z)$ (*lower panel*) for $(\Omega_0,\, \lambda_0) = (1.0,0.0)$ in *dashed line*, (1.0,0.9) in *dot-dashed line*, (0.1,0.9) in *dotted line*, and (1.0,0.9) in *thick solid line.*

Figure 2 : The contours of $\xi^{(s)}(s_{\perp}, s_{\parallel})$ in CDM models at $z = 0$ (*upper panels*) and $z = 3$ (*lower panels*). $H_0 = 70\mathrm{km{\cdot}s^{-1}{\cdot}Mpc^{-1}}$ is assumed, and the amplitude of the fluctuation spectrum is normalized according to the COBE 2 yr data. Solid and dashed lines indicate the positive and negative $\xi^{(s)}$, respectively. Contour spacings are $\Delta\log_{10}|\xi| = 0.25$. Thick lines in all $\Omega_0 = 1$ models represent $\xi^{(s)} = 10$, 1, 0.1, -0.1, and -1. Thick lines in $\Omega_0 = 0.1$ models indicate that $\xi^{(s)} = 0.01$ and 0.001 for $\lambda_0 = 0$ at $z = 0$, $\xi^{(s)} = 1$, 0.1 and 0.01 for $\lambda_0 = 0.9$ at $z = 0$, $\xi^{(s)} = 0.01$ for $\lambda_0 = 0$ at $z = 3$, and $\xi^{(s)} = 0.1$ for $\lambda_0 = 0.9$ at $z = 3$.

Figure 3 : The anisotropy parameter $\xi_{\parallel}^{(s)}(s)/\xi_{\perp}^{(s)}(s)$ as a function of z. Upper and lower panels assume that $b = 1$ and 2, respectively. From left to right, the panel corresponds to CDM at $s = 10h^{-1}$Mpc, CDM at $s = 20h^{-1}$Mpc, a power-law model with $\gamma = 1.8$ and a power-law model with $\gamma = 1.0$.


Ω_0 = 1.0, λ_0 = 0.0
Ω_0 = 1.0, λ_0 = 0.9
Ω_0 = 0.1, λ_0 = 0.0
Ω_0 = 0.1, λ_0 = 0.9
$\eta(z) = c_\parallel(z)/c_\perp(z)$
$f(z) = \beta(z)\ b(z)$
redshift z
0
0.5
1
0
2
4
6


CDM models
Ω_0 = 1.0, λ_0 = 0.0
Ω_0 = 1.0, λ_0 = 0.9
Ω_0 = 0.1, λ_0 = 0.0
Ω_0 = 0.1, λ_0 = 0.9
$s_\parallel$ (h^{-1}Mpc)
$s_\perp$(h^{-1}Mpc)
z = 0
z = 3
10
0.01
1
0.1



CDM, s=10h^{-1}Mpc
CDM, s=20h^{-1}Mpc
γ= 1.8
γ= 1.0
b = 1
b = 2
Ω_0 λ_0
1.0 0.0
1.0 0.9
0.1 0.0
0.1 0.9
$\xi_{\parallel}^{(s)}/\xi_{\perp}^{(s)}$
redshift z